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OPTIMIZATION OF THE STRUCTURE OF A VIBRATION SHIELD UNDER THE INFLUENCE OF A CONCENTRATED HARMONIC LOAD

K. S. Adamova and M. A. Kasnibolotskii

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When waves strike the interface between media with different physicomechanical properties, a system of reflected and refracted waves is formed in the laminated medium. By changing the number, size, and material of the layers, it is possible to control the intensity of the spectrum of the wave process. There naturally arises the problem of optimizing the structure of the laminated medium with different optimization criteria and different constraints on the characteristics of the wave process. Several studies [1-5] have examined aspects of optimization of the structure of multilayered sound-reflecting shields when the materials of the layers are chosen from a certain group. Investigators have examined both the case of normal incidence of an acoustic plane wave and oblique incidence. If neither the number nor the arrangement of the constituent materials is specified beforehand, then the optimization problem is formulated within the framework of the theory of optimum control. Pontryagin's maximum principle and variational methods have been used to derive the necessary optimization conditions and construct algorithms for numerical calculations. The same methods, generalized in [5], have also been used to optimize the design of a freely oscillating laminated thick-walled sphere of minimum weight [6], in several problems involving the static thermoelasticity of thick-walled spherical vessels [7, 8], and in the design of laminated thermal insulation [5, 9, 10] and wave-type electromagnetic filters [2]. In each of these studies, the spectral characteristics of the wave process depended on one space variable and were described by ordinary differential equations.

In the present study, we examine the steady vibration of a plane elastic laminated shield which is rigidly connected to an elastic half-space and is subjected to a concentrated harmonic load. We need to optimize the structure of the shield so as to minimize total wave-energy flux in the half-space. The spectral characteristics of the wave process will depend on two space variables and will be described by partial differential equations. By using the Hankel transform [11] with respect to the radial coordinate, it is possible to formulate the corresponding optimization problem for transforms that can be described by a system of ordinary differential equations. We obtain the necessary optimization conditions, propose an algorithm, and present examples of numerical calculations.

1. Formulation of the Problem. We will examine the steady-state vibration of an elastic laminated shield of thickness $l > 0$. The shield is rigidly connected to an elastic half-space $z > l$, and is subjected to a concentrated harmonic force (see Fig. 1). Choosing from a finite number of elastic materials, we need to synthesize a laminated shield occupying the region $0 \le z \le l$. The shield must be designed so as to minimize the total energy

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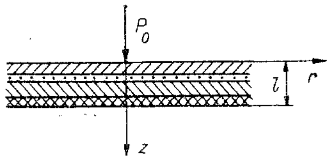

Fig. 1

flux in the half-space $z > l$. The formulation of this optimization problem in terms of optimum control theory involves description of the control system, the set of control variables, the functionals in the minimization criterion, and the constraints.

The equations of the physical process will serve as the system being controlled. In the given case, these are the equations of steady vibration of the laminated half-space. Written in a cylindrical coordinate system at $r > 0$, $0 \le z \le \infty$, these equations take the following form [11] for the case of axial symmetry {the multiplier $\exp(i\omega t)$ is omitted]

$$\frac{\partial G^+}{\partial r} + \frac{\partial \Gamma^+}{\partial z} + \frac{2\mu}{\lambda}\frac{\partial}{\partial r}\left(G^- - 2\mu\frac{\partial U_z}{\partial z}\right) + \rho\omega^2 U_r = 0,$$
$$\frac{\partial G^+}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(\lambda r\Gamma^+) + \frac{2}{r}\frac{\partial}{\partial r}(r\Gamma^-) + \rho\omega^2 U_z = 0, \tag{1.1}$$

where

$$G^\pm = 2\mu\frac{\partial U_z}{\partial z} + \lambda\left[\frac{1}{r}\frac{\partial}{\partial r}(rU_r) \pm \frac{\partial U_z}{\partial z}\right]; \quad \Gamma^\pm = \mu\left(\frac{\partial U_r}{\partial z} \pm \frac{\partial U_z}{\partial r}\right).$$

The conditions on the boundaries of the layers at $z > 0$ follow from the requirements of continuity of the complex amplitudes of the components U_r and U_z of the displacement vector and the continuity of the normal $\sigma_{zz} = G^+$ and shear $\sigma_{zr} = \Gamma^+$ components of the stress vector:

$$[U_r] = [U_z] = [G^+] = [\Gamma^+] = 0. \tag{1.2}$$

Since we are examining a surface source in the form of a normal concentrated force $\mathrm{Re}[f(r)\cdot \exp(i\omega t)]$, we write the boundary conditions at $z = 0$ in the form [11]

$$G^+(r, 0) = f(r) = \frac{P_0}{2\pi}\frac{\delta(r)}{r}, \quad \Gamma^+(r, 0) = 0. \tag{1.3}$$

The Lamé constants λ and μ in (1.1) are piecewise-constant functions of the coordinate z. Boundary-value problem (1.1)-(1.3) is augmented by the condition of radiation at infinity [12].

The set of control variables in the problem is the set of all possible laminated structures of thickness l, These structures can be composed of the given set of initial materials. We proceed as follows to describe this set. We place each material in correspondence with the serial number indicating its position in the set. We introduce the characteristic function of the laminated medium $u(z)$. At each point $z \in [0, l]$, this function takes a whole-number value equal to the serial number of the material located at the given point. The function $u(z)$ belongs to the class of piecewise-constant functions

$$u(z) = \{u_s | z_s < z \leqslant z_{s+1}\},\ s = 1, \dots, I;\ z_1 = 0,\ z_{I+1} = l, \tag{1.4}$$

the range of values of these functions consisting of whole numbers from 1 to m

$$u_s \in \{1, \dots, m\} = \Lambda, \tag{1.5}$$

where z_s ($s = 2, \dots, I$) are the interfaces between the layers; I is the number of layers; m is the number of initial materials. Since a one-to-one correspondence can be established between the set of laminated structures and the set of functions $u(z)$, we choose the characteristic function of the laminated medium (1.4)-(1.5) as the control function. The domain of $u(z)$ unambiguously determines the number, size, and arrangement of the materials of the layers. It is evident that $\lambda = \lambda[u(z)]$, $\mu = \mu[u(z)]$, $\rho = \rho[u(z)]$.

As the quantity to be minimized, we examined the total energy flux across the plane $z = l$ [12]:

$$F(u) = \pi\omega \int_0^\infty r \operatorname{Im} [\sigma_{zz}(r, l) \overline{U}_z(r, l) + \sigma_{zr}(r, l) \overline{U}_r(r, l)]\, dr \tag{1.6}$$

(the superimposed bar denotes complex conjugation).

The optimization problem is formulated mathematically as follows: from among the functions (1.4)-(1.5) given on the interval $[0, l]$, find the function $u^{opt}(z)$ that minimizes the functional (1.6); the functions $U_r(r, z)$, $U_z(r, z)$, $\sigma_{zr}(r, z)$, and $\sigma_{zz}(r, z)$ which enter into the functional are determined from the solution of boundary-value problem (1.1)-(1.3).

2. Reduction of Problem (1.1)-(1.6) to a Canonical Optimum-Control Problem. It is known that the difficulty encountered in solving optimization problems increases dramatically with an increase in the number of independent variables. On the other hand, the sought control function $u(z)$ depends only on one space variable z. It is thus natural to attempt to use the Hankel transform to effect a "convolution" of the radial coordinate r and replace (1.1)-(1.3) by a system of ordinary differential equations in transforms of the original independent variables. Representing the solution U_r, U_z in the form

$$U_r(r, z) = \int_0^\infty J_1(\alpha r) P(\alpha, z)\, d\alpha,\quad U_z(r, z) = \int_0^\infty J_0(\alpha r) S(\alpha, z)\, d\alpha \tag{2.1}$$

(J_0 and J_1 are Bessel functions) and having inserted it into (1.1), we obtain the following second-order system of ordinary differential equations for the transforms P and S [11]:

$$\begin{gathered}[\mu(P' - \alpha S)]' - \alpha\lambda S' - (\lambda + 2\mu)\eta^2 P = 0,\\ [(\lambda + 2\mu)S' + \lambda\alpha P]' + \alpha\mu P' - \mu\xi^2 S = 0.\end{gathered} \tag{2.2}$$

Here, $\eta^2 = \alpha^2 - K_P^2$; $\xi^2 = \alpha^2 - K_S^2$; $K_p = \omega/[(\lambda + 2\mu)/\rho]^{1/2}$; $K_S = \omega/(\mu/\rho)^{1/2}$; the primes denote differentiation with respect to z. As a result of the transformation of (1.3), we find the boundary conditions for the transforms P and S at $z = 0$:

$$\begin{gathered}(\lambda + 2\mu)S' + \alpha\lambda P = f_0,\ P' - \alpha S = 0,\ z = 0\\ \left(f_0(\alpha) = \alpha \int_0^\infty J_0(\alpha r) f(r)\, r\, dr = P_0\alpha/(4\pi)\right).\end{gathered} \tag{2.3}$$

We introduce the new variables $y_1 = -P/\alpha^2$, $y_2 = S/\alpha$, $y_3 = -\mu(P' - \alpha S)/\alpha^2$, $y_4 = [(\lambda + 2\mu)S' + \alpha\lambda P]/\alpha$, which by virtue of (1.2) remain continuous with the transition through the interfaces. It follows from (2.2), (2.3) that the vector $\mathbf{y} = \{y_1, \ldots, y_4\}$ satisfies the first-order system

$$\mathbf{y}'(\alpha, z) = A(\alpha, u)\mathbf{y}(\alpha, z); \tag{2.4}$$

$$y_3(\alpha, 0) = 0,\ y_4(\alpha, 0) = P_0/(4\pi), \tag{2.5}$$

where $A = \|a_{ij}\|$ $(i, j = 1, \ldots, 4)$; $a_{11} = a_{14} = a_{22} = a_{23} = a_{32} = a_{33} = a_{41} = a_{44} = 0$; $a_{12} = -1$; $a_{13} = \mu^{-1}$; $a_{21} = \alpha\ (+ 2\mu)^{-1}$; ${}^2\lambda(\lambda + 2\mu)^{-1}$; $a_{24} = (\lambda + 2\mu)^{-1}$; $a_{31} = (\lambda + 2\mu)\eta^2 - (\alpha\lambda)^2(\lambda + 2\mu)^{-1}$; $a_{34} = -\lambda(\lambda + 2\mu)^{-1}$; $a_{42} = -\mu(\alpha^2 - \xi^2)$; $a_{43} = \alpha^2$. Due to the continuity of the vector $\mathbf{y}$, system (2.4) is valid over the entire semi-infinite interval $0 \le z < \infty$. To close it, we supplement the boundary conditions at $z = 0$ with conditions of radiation at infinity. Equations (2.4) have the following properties: 1) the elements of matrix A are determined at $z \ge 0$, depend on the initial equation of $u(z)$, and are piecewise-constant functions; 2) Eq. (2.4) establishes a one-parameter family of solutions dependent on the transformation parameter α. Using the procedure described in [5], we can reduce the effect of the half-space $z > l$ on the wave pattern in the laminated shield $0 \le z \le l$ to two boundary conditions at $z = l$. The form of the solution will differ, depending on to which of the three regions the parameter α belongs: $0 \le \alpha < K_P$ or $K_P \le \alpha < K_S$ or $K_S \le \alpha < \infty$. For example, for the first case

$$y_1(\alpha, z) = A_P \exp[i\gamma_1(z - l)] + i\gamma_2 A_S \exp[i\gamma_2(z - l)],$$

$$y_2(\alpha, z) = i\gamma_1 A_P \exp[i\gamma_1(z-l)] + \alpha^2 A_S \exp[i\gamma_2(z-l)],$$
$$y_3(\alpha, z) = \mu\{2i\gamma_1 A_P \exp[i\gamma_1(z-l)] + \beta A_S \exp[i\gamma_2(z-l)]\},$$
$$y_4(\alpha, z) = \mu\{\beta A_P \exp[i\gamma_1(z-l)] + 2i\alpha^2\gamma_2 A_S \exp[i\gamma_2(z-l)]\}. \quad (2.6)$$

Here, $\gamma_1^2 = K_P^2 - \alpha^2$; $\gamma_2^2 = K_S^2 - \alpha^2$; $\beta = 2\alpha^2 - K_S^2$; A_P, A_S are unknown arbitrary constants. Having the solution in the form (2.6) ensures satisfaction of the conditions of radiation at infinity. If solution (2.6) is written for $z = l$, if A_P and A_S are expressed through the first two equations, and if we insert these quantities into the third and fourth equations, we obtain the boundary conditions

$$y_3(\alpha, l) = g_{11}y_1(\alpha, l) + g_{12}y_2(\alpha, l),$$
$$y_4(\alpha, l) = g_{21}y_1(\alpha, l) + g_{22}y_2(\alpha, l), \quad (2.7)$$

where $g_{11} = i\mu\gamma_1 K_S^2\Delta$; $g_{12} = \mu(\beta + 2\gamma_1\gamma_2)\Delta$; $g_{21} = \alpha^2 g_{12}$; $g_{22} = i\mu\gamma_2 K_S^2\Delta$; $\Delta = (\alpha^2 + \gamma_1\gamma_2)^{-1}$. When $K_P \le \alpha < K_S$, the boundary conditions at $z = l$ have the same form (2.7) but with the following values g_{ij} $(i, j = 1, 2)$: $g_{11} = -i\mu\eta K_S^2\Delta_1$, $g_{12} = \mu(\beta + 2i\gamma_1\eta)\Delta_1$, $g_{21} = \alpha^2 g_{12}$, $g_{22} = i\mu\gamma_1 K_S^2\Delta_1$, $\Delta_1 = (\alpha^2 + i\gamma_1\eta)^{-1}$. It can be seen from further considerations that solutions (2.4) are not used for $\alpha > K_S$. Thus, the value of $\mathbf{y}(\alpha, z)$ on the segment $0 \le z \le l$ is found from the solution of boundary-value problem (2.4)-(2.5), (2.7).

To finally formulate the optimization problem in terms of transforms, we express them through the functional (1.6) being minimized. To do this, we need to use the transformation formulas to write the following initial quantities

$$U_r(r, z) = -\int_0^\infty \alpha^2 J_1(\alpha r)\, y_1(\alpha, z)\, d\alpha,\ U_z(r, z) = \int_0^\infty \alpha J_0(\alpha r)\, y_2(\alpha, z)\, d\alpha,$$
$$\sigma_{zr}(r, z) = -\int_0^\infty \alpha^2 J_0(\alpha r)\, y_3(\alpha, z)\, d\alpha,\ \sigma_{zz}(r, z) = \int_0^\infty \alpha J_0(\alpha r)\, y_4(\alpha, z)\, d\alpha \quad (2.8)$$

and insert them into integral (1.6). Then, with allowance for the Parseval equality, the minimizing functional is written as

$$F(u) = \pi\omega\, \mathrm{Im}\int_0^\infty \alpha\left[\bar{y}_2(\alpha, l)\, y_4(\alpha, l) + \alpha^2\bar{y}_1(\alpha, l)\, y_3(\alpha, l)\right] d\alpha. \quad (2.9)$$

Direct numerical integration of (2.8)-(2.9) is impossible due to the fact that the integrands contain a finite number of poles α_f, which are zeros of the Rayleigh denominator $R(\alpha) = 0$ [11, 12]. The presence of the poles is connected with the formation of Rayleigh and Lamb waves on the surface $z = 0$ and the boundary layers. The number of poles and the values of α_f generally depend on the structure of the laminated medium and frequency. It can be shown that

$$\mathrm{Im}\int_{K_S^0}^\infty \alpha\left[\bar{y}_2(\alpha, l)\, y_4(\alpha, l) + \alpha^2\bar{y}_1(\alpha, l)\, y_3(\alpha, l)\right] d\alpha = 0.$$

Here, $K_S^0 = \omega/(\mu^0/\rho^0)^{1/2}$ is the second wave number for the elastic half-space $z > l$. It follows from this that the minimizing functional has a definite integral

$$F(u) = \pi\omega\, \mathrm{Im}\int_0^{K_S^0} \alpha\left[\bar{y}_2(\alpha, l)\, y_4(\alpha, l) + \alpha^2\bar{y}_1(\alpha, l)\, y_3(\alpha, l)\right] d\alpha \quad (2.10)$$

with an integrand that does not contain poles. We finally formulate the original optimization problem in canonical Pontryagin form: from among piecewise-constant functions (1.4) with a whole-number range of values (1.5), find $u^{opt}(z)$ $(0 \le z \le l)$ minimizing the functional (2.10). The functions $y_i(\alpha, l)$, $(i = 1, \ldots, 4)$ entering into the integrand are found for each $0 \le \alpha \le K_S^0$ from the solution of boundary-value problem (2.4)-(2.5), (2.7).

3. Necessary Optimality Conditions. To deduce these conditions, it is necessary to construct a variation of functional (2.10) generated by variation of the control function u(z). Classical methods of variational calculus cannot do this, since – by virtue of (1.5) – the control function u(z) does not have infinitesimals in a uniform norm of variations. By the term "perturbed control function," we will mean the function [13]

$$u^*(z) = \begin{cases} \vartheta; & z \in M, \ \vartheta \in \Lambda, \\ u; & z \in [0, l] \setminus M, \end{cases} \tag{3.1}$$

where $M \subset [0, l]$ is a set of small measure; mes $M = \varepsilon \ll l$; $\varepsilon > 0$ is an infinitesimal of the first order. The principal increment of functional (2.10), generated by needle variation $\{M, \vartheta\}$, has the form

$$\delta F(M, \vartheta) = \int_M [H(\mathbf{y}, \boldsymbol{\psi}, u) - H(\mathbf{y}, \boldsymbol{\psi}, \vartheta)]\, dz. \tag{3.2}$$

The Hamiltonian

$$\begin{aligned} H(\mathbf{y}, \boldsymbol{\psi}, u) = \operatorname{Re} \int_0^{K_S^0} \Big[& \frac{1}{\mu} y_3\psi_1 + \frac{1}{\lambda+2\mu} y_4\psi_2 + \frac{2\mu}{\lambda+2\mu}(y_4\psi_3 - \\ & - \alpha^2 y_1\psi_2) - \rho\omega^2 (y_1\psi_3 + y_2\psi_4) + \alpha^2 \frac{4\mu(\lambda+\mu)}{\lambda+2\mu} y_1\psi_3 + \\ & + \alpha^2 (y_1\psi_3 + y_3\psi_4) - (y_2\psi_1 + y_4\psi_3) \Big] d\alpha. \end{aligned} \tag{3.3}$$

The conjugate vector function $\boldsymbol{\psi} = \{\psi_i\}$ $(i = 1, \ldots, 4)$ is found from the solution of the conjugate boundary-value problem

$$\begin{gathered} \boldsymbol{\psi}'(\alpha, z) = -A^*(\alpha, u)\boldsymbol{\psi}(\alpha, z), \\ \psi_1(\alpha, 0) = 0, \ \psi_1(\alpha, l) + g_{11}\psi_3(\alpha, l) + g_{21}\psi_4(\alpha, l) = 2\pi\omega\alpha^3 Q_1, \\ \psi_2(\alpha, 0) = 0, \ \psi_2(\alpha, l) + g_{12}\psi_3(\alpha, l) + g_{22}\psi_4(\alpha, l) = 2\pi\omega\alpha Q_2. \end{gathered} \tag{3.4}$$

Here

$$Q_1 = \begin{cases} \operatorname{Im}(g_{11})\,\bar{y}_1(\alpha, l), & 0 \leqslant \alpha \leqslant K_P^0, \\ \operatorname{Im}(g_{11})\,\bar{y}_1(\alpha, l) + \operatorname{Im}(g_{12})\,\bar{y}_2(\alpha, l), & K_P^0 < \alpha \leqslant K_S^0; \end{cases}$$

$$Q_2 = \begin{cases} \operatorname{Im}(g_{22})\,\bar{y}_2(\alpha, l), & 0 \leqslant \alpha \leqslant K_P^0, \\ \operatorname{Im}(g_{21})\,\bar{y}_1(\alpha, l) + \operatorname{Im}(g_{22})\,\bar{y}_2(\alpha, l), & K_P^0 < \alpha \leqslant K_S^0. \end{cases}$$

If $u(z) = u^{\mathrm{opt}}(z)$, then $\delta F \geq 0$ for all M and $\vartheta \in \Lambda$. Following from this are the necessary optimality conditions in the form of Pontryagin's maximum principle: let $u^{\mathrm{opt}}(z)$ be an optimum control function minimizing functional (2.10), and let $\mathbf{y}(\alpha, z)$, $0 \leq \alpha \leq K_S{}^0$, $0 \leq z \leq l$ be the corresponding family of solutions of boundary-value problem (2.4)-(2.5), (2.7). Then there exists a family of solutions $\boldsymbol{\psi}(\alpha, z)$ of conjugate boundary-value problem (3.4) such that the Hamiltonian (3.3) constructed with it reaches its maximum value with respect to the argument u of the optimal control function for almost any $z \in [0, l]$:

$$H\left(\mathbf{y}, \boldsymbol{\psi}, u^{\mathrm{opt}}\right) = \max_{u \in \Lambda} H(\mathbf{y}, \boldsymbol{\psi}, u). \tag{3.5}$$

4. Computational Algorithm. The computational procedure used to find the optimum solution consists of constructing a minimizing sequence of control functions $u^n(z)$, n = 1, 2, ... The transition to the next approximation is made by selecting a set of small measure M and a value $\vartheta \in \Lambda$ on this set for which the perturbed control function (3.1) reduces the functional (2.10). There are several algorithms for constructing a minimizing sequence, these algorithms differing in the method used to assign the set M. We will describe one of the most efficient variants [14]. Assume that we know the current approximation $u^n(z)$ and the corresponding solutions $\mathbf{y}^n(\alpha, z)$ and $\boldsymbol{\psi}^n(\alpha, z)$ of the direct and conjugate problems. We subdivide the segment $[0, l]$ into a sufficiently large number of equal intervals $\Delta z_i = l/N = \Delta z$ $(i = 1, \ldots, N)$, $\Delta z_i = z_{i+1} - z_i$. We determine $\hat{u}(z)$ and $w(\hat{u}, z)$:

TABLE 1

No. of material	$\mu\cdot 10^{-10}$	$\lambda\cdot 10^{-10}$	$\rho\cdot 10^{-3}$	No. of material	$\mu\cdot 10^{-10}$	$\lambda\cdot 10^{-10}$	$\rho\cdot 10^{-3}$
1	0,145	1,15	1,20	4	2,50	16,00	2,70
2	2,50	12,22	2,30	5	2,66	14,80	2,50
3	3,90	16,6	2,65	6	2,73	12,30	2,21

$$\widehat{u}(z) = \arg\max_{u\in\Lambda} H(\mathbf{y}^n, \psi^n, u),$$
$$w(\widehat{u}, z) = H(\mathbf{y}^n, \psi^n, \widehat{u}) - H(\mathbf{y}^n, \psi^n, u^n) \geqslant 0.$$

It is evident that the variation of functional (3.2) can be written in the form

$$\delta F = -\int_M w(\widehat{u}, z)\, dz \leqslant 0.$$

If u^n is nonoptimal, then we can find an M small enough so that the variation of δF will become less than zero. We proceed as follows in order to best choose the set M. We designate $w_i = w[\hat{u}(z_i + \Delta z/2), z_i + \Delta z/2]$ ($i = 1, \ldots, N$). We arrange the values of w_i in descending order. As a result, we obtain the set w_{j_k}, $k = 1, \ldots, N$ ($w_{j_{k+1}} \leq w_{j_k}$), where j_k denotes the number of the interval. We form the sets M_i ($i = 1, \ldots, N$) by the rule

$$M_i = \bigcup_{k=1}^{i} \Delta z_{j_k}$$

(obviously, mes $M_1 = \Delta z$, mes $M_N = l$) and we introduce

$$u_i^n(z) = \begin{cases} \widehat{u}; & z \in M_i, \\ u^n; & z \notin M_i. \end{cases}$$

We designate $i^0 = \arg\min_i F(u_i{}^n)$. Then $u^{n+1}(z) = u_{i^0}{}^n(z)$. We choose $u^{n+1}(z)$ as the new control function. We calculate the corresponding $\mathbf{y}^{n+1}, \psi^{n+1}$, for it, construct the new Hamiltonians, find $\hat{u}$ and w, and so forth. The process ends when $w_i = 0$, $i = 1, \ldots, N$. This condition is equivalent to satisfaction of (3.5).

5. Sample Calculations. Choosing from among the materials shown in Table 1, we need to synthesize a nonuniform shield, with a total thickness $l = 0.1$ m, that will minimize the total energy flux in the elastic half-space $z > l$, from a concentrated harmonic force of unit amplitude and the frequency $\omega = 2\pi$ 2000 Hz. The properties of the elastic half-space are assigned: $\lambda = 10.66\cdot 10^9$ N/m², $\mu = 1.83\cdot 10^9$ N/m², $\rho = 917$ kgf/m³. In calculating the Hamiltonian (3.3), we subdivided the interval $[0, K_S{}^0]$ of integration over α into 10, 15, and 20 parts and replaced the integral by the corresponding partial sums. Given such subdivisions, the values of the test functional differed by no more than 2%. The calculations revealed that the optimum shield was a two-layer shield: the first layer, with a thickness of $2.4\cdot 10^{-2}$, is made of the first material in the table; the second layer, with a thickness of $7.6\cdot 10^{-2}$ m, is made of the fourth material. The efficiency of the optimum shield is evaluated by the quantity $\theta = F^{opt}/F_0$, where F^{opt} is the total energy flux in the half-space $z > l$ in the direction of the z axis when the optimum shield is present; F_0 is the same in the absence of the shield. For the example we have calculated, $\theta = 0.268$. If the concentrated force in our example acts with the frequency $\omega = 2\pi$ 5000 Hz, then the optimum shield consists of four layers: the first, $1.6\cdot 10^{-2}$ m thick, is made of the first material in the table; the second, $2.5\cdot 10^{-2}$ m thick, is made of the fourth material; the third, $4.4\cdot 10^{-2}$ m thick, is made of the third material; the fourth, $1.5\cdot 10^{-2}$ m thick, is made of the fourth material. The efficiency of this shield is equal to $\theta = 0.099$.

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PLASTIC DEFORMATION OF AN ISOTROPICALLY STRAIN-HARDENING POLYCRYSTALLINE MATERIAL

O. A. Volokhovskaya and V. V. Podalkov

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It is known that the features of the elastoplastic deformation of metals are determined by their polycrystalline structure. Thus, the equations that describe the deformation of the polycrystal should be derived on the basis of study of the processes that take place within its grains. It has been established experimentally that, at moderate temperatures, plastic deformation occurs mainly by the mechanism of translational crystallographic slip. Slip is anisotropic and leads to strain-hardening of a single crystal. This strain-hardening is expressed in an increase in the limiting shear stress in both active (active strain-hardening) and passive (latent strain-hardening) systems and must be taken into account when choosing the corresponding strain-hardening law. The elastic and plastic anisotropy of crystallites and intergranular interactions occurring throughout the deformation history of the material cause the fields of local stresses and strains in it to be nonuniform. Thus, the equation that connects macroscopic stresses and strains should be determined by averaging relations between the corresponding local fields over the entire volume of the specimen. There is a fairly large number of consistent theories in which the researcher, in the course of deriving the governing equation, made a detailed study of one of the above-mentioned aspects of the plastic deformation of polycrystals [1-5].

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